SECOND ORDER NONLINEARITIES AND THEIR APPLICATION IN NDE

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INTRODUCTION AND BACKGROUND

Although it has been recognized for quite some time that solid materials do not strictly obey Hooke's law, linear elastic wave studies provided valuable tools to obtain both microscopic and macroscopic material parameters. Ultrasonic wave velocities can be used to obtain second order elastic constants while attenuation measurements can be related to many parameters, e.g. grain size in metals. It is quite recent that the NDE community turned its attention to investigate the possibility of using nonlinear acoustic techniques to measure properties of solids and interfaces [1-8]. The so-called nonlinearity parameter which is the combination of second and third order elastic constants has been extensively studied for crystalline solids [1] and has been related to microscopic behavior of the solid, e.g. a harmonic behavior of the interatomic potential as well as to macroscopic quantities e.g., thermal expansion, residual stress and material hardness. Basically, there are two approaches to obtain third order elastic constants and subsequently the so-called nonlinearity parameter from ultrasonic measurements. Either from the second harmonic generation [1] of a finite amplitude elastic wave or from the so-called acousto-elastic [9] effect, i.e. from stress dependent velocity data. An extensive list is available for the third order elastic constants in the literature [10]. There are several indications that higher than third order elastic constants could play a role in material characterization, e.g. fourth and fifth order elastic constants enter in the temperature dependence of velocity as functions of stress [11,12]. It has been suggested also that higher order nonlinearities may be more strength related in an adhesive joint than linear parameters [5]. There are practically no reported measurements on fourth order elastic constants available although theoretical considerations show that there are 4 fourth order elastic constants which describe an isotropic solid. It is not evident how harmonic generation could be developed for fourth order elastic constant measurement. On the other hand, the stress dependence of the velocity when carried out up to a second order offers several possibilities for measuring second order nonlinearities, and hence perhaps fourth order elastic constants. The sound velocity in a stressed solid may be expressed

$$C(\sigma) = C_0 + C_1\sigma + C_2\sigma^2 + \dots \qquad (1)$$

where C_0 is the velocity in the unstressed material. C_1 is actually the first order acousto-elastic constant and is a combination of the second and third order elastic constants from which first order nonlinearity parameters can be obtained. C_2 may be called a second order acousto-elastic constant and it is given as the combination of second, third, and fourth order elastic constants and related to second order nonlinearity parameters. Depending on the direction of σ with respect to wave propagation and polarization direction, both C_1 and C_2 have different forms. Methods to determine C_1 from acousto-elastic measurement are well documented in the literature. In order to obtain C_2 from Eq. 1 one can use extremely high shock compression stresses as was done for fused quartz and sapphire [13]. This approach however is clearly not applicable to many NDE applications. The possibility of measuring simultaneously both C_1 and C_2 at relatively low stress levels (below 15% of the yield) was reported last year [5] using a dynamic acousto-elastic measurement technique. In this technique a high amplitude, low frequency external excitation provides stresses in an adhesive bond. A high frequency broadband shear wave transducer in a pulse-echo mode excites waves in the bond. The recorded velocities are measured as functions of first and second harmonic of the external load thus providing both stress and stress square dependence of the velocities. In this paper we are proposing two new measurement techniques to provide second order nonlinearity parameters. Both methods are based on the elimination of the first order term in the stress dependence of the velocity; therefore, any nonlinear effect measured will be due to a second (or higher) order. The first method eliminates the linear stress dependent term by utilizing the effect of "acoustic birefringence" at a proper polarization angle of the shear wave and we call it polarization technique. The second method is a modified acousto-elastic measurement by applying a "pure" shear stress to the sample. It was pointed out before [5] that due to symmetry considerations the lowest order nonlinear term appears only as the stress square term with C_2 as coefficient. This second method is referred to as "pure" shear stress technique.

POLARIZATION TECHNIQUE

Theoretical Considerations

When an isotropic solid is prestressed under uniaxial tension or compression a slight anisotropy is introduced in the material. As the result of this anisotropy, the velocity of the shear wave will depend on the polarization direction with respect to the applied stress. This phenomenon is known as "acoustic birefringence." By designating the stress dependent shear wave velocity with parallel polarization to the applied stress $C'(\sigma)$ and the shear wave velocity with perpendicular polarization to the applied stress by $C''(\sigma)$, Eq. 1 may be written as

$$C'(\sigma) = C_0' + C_1'\sigma + C_2'\sigma^2 \qquad (2)$$

and

$$C''(\sigma) = C_0'' + C_1''\sigma + C_2''\sigma^2 \qquad (3)$$

In Eqs. 2 and 3 it is assumed that the shear velocities are not the same at both polarization directions at zero stress, i.e. there is some texture in the material. Now consider an ultrasonic shear wave polarized in an arbitrary angle θ with respect to the axis of the applied stress (Fig. 1).

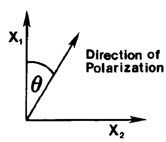

Fig. 1. Schematic diagram of a shear wave polarized in an arbitrary angle.

The received birefringent signal may be written as

$$u = \cos^2\Theta\cdot\exp[i\omega\cdot 2D/c'(\sigma)] + \sin^2\Theta\cdot\exp[i\omega\cdot 2D/c''(\sigma)] \tag{4}$$

where D is the sample thickness. Here, the amplitude variation of the two signals is expressed only in terms of the direction of polarization. In order to measure the weak second order effect at moderate stress levels, an optimal angle for shear wave polarization is selected where the stronger first order effect can be reduced or eliminated. At this angle the velocity may be written as

$$C_\Theta(\sigma) = C_{\Theta 0} + C_{\Theta 2}\sigma^2 \tag{5}$$

and the received signal at that angle is

$$u_\Theta \simeq \exp[i\omega\cdot 2D/C_\Theta(\sigma)] \ . \tag{6}$$

When the phase difference of the two signals is small, i.e. when

$$\omega[2D/c'(\sigma) - 2D/c''(\sigma)] << 1 \tag{7}$$

then the phase terms in Eq. 4 can be expanded and equated to the phase term in Eq. 6 and one obtains

$$C_{\Theta 0} + C_{\Theta 2}\sigma^2 = C_0'\cos^2\Theta + C_0''\sin^2\Theta + \sigma[C_1'\cos^2\Theta + C_1''\sin^2\Theta] + \sigma^2[C_2'\cos^2\Theta + C_2''\sin^2\Theta] \tag{8}$$

from the second term of Eq. 8, one obtains the condition for the polarization angle when the first order term vanishes

$$C_1'\cos^2\Theta + C_1''\sin^2\Theta = 0 \tag{9}$$

from Eq. 8 one obtains

$$\tan\Theta = \sqrt{-C_1'/C_1''} \tag{10}$$

Equation 10 implies that the required condition to find a polarization angle Θ is that the slope of the two stress dependent velocities has to be of opposite sign. The zeroth and second order coefficients from Eqs. 5 and 8 can be given as

$$C_{\Theta 0} = C_0'\cos^2\Theta + C_0''\sin^2\Theta \tag{11}$$

and

$$C_{\Theta 2} = C_2'\cos^2\Theta + C_2''\sin^2\Theta . \tag{12}$$

The experimental objective is to find an angle of polarization predicted by Eq. 10.

Experimental Considerations

The schematic of a standard acousto-elastic measurement system is shown in Fig. 2. An ultrasonic contact transducer at 5 MHz (either longitudinal or shear) is coupled to the aluminum block. The transducer is operated in pulse-echo mode and the received echoes are displayed on a LeCroy 9400 digital oscilloscope. The aluminum block was loaded in uniaxial compression and tension ranging from -19 - 11 KN in increments of 1KN. The ultrasonic velocity was measured as a function of applied stress. Utilizing the digital capabilities of the oscilloscope the time resolution of the first backwall echo was increased to .02 nanoseconds. When an external stress is applied to the sample a change in the velocity occurs and as a consequence the time shift of the received signal will be proportional to the applied stress. In Fig. 3 the longitudinal velocity is plotted as a function of the applied compressive and tension stress. From the "best fit" linear regression line we obtain the first acousto-elastic constant for the longitudinal waves

$C_1''' = 123 \text{ m s}^{-1} \text{ ksi}^{-1}$.

In Fig. 4 shear wave velocity for the case of parallel and perpendicular polarization is given as the function of applied stress. The corresponding first order acousto-elastic constants are obtained as

$C_1' = -\ 0.459 \text{ m s}^{-1} \text{ ksi}^{-1}$

and

$C_1'' = 0.129 \text{ m s}^{-1} \text{ ksi}^{-1}$.

From Fig. 4 it should be noticed that at zero stress the two shear velocities are not the same indicating some texture in the material. In order to measure the second order acousto-elastic coefficient $C_{\Theta 2}$ the polarization angle was determined from Eq. 9 by substituting the measured values of C' and C". The polarization angle at which the first order effect is minimized for this aluminum is 28°. The shear wave velocity with 28° polarization angle was measured as the applied stress in the aluminum sample. The experimental result is plotted in Fig. 5. A second order polynomial "best fit" routine was used to determine the relationship between the stress dependent shear wave velocity and the applied stress resulting in the following equation:

$$C_\Theta(\sigma) = 3174 - 0.00155\sigma^2 \qquad (13)$$

where 3174 m/sec is the zero stress shear velocity with 28 polarization angle relative to the applied stress. The second order acousto-elastic coefficient is $C_{\Theta 2} = 0.00155 \text{ m s}^{-1}\text{ksi}^{-2}$, and only 1% of the first order term. The coefficient $C_{\Theta 2}$ is a combination of second, third, and fourth order elastic constants.

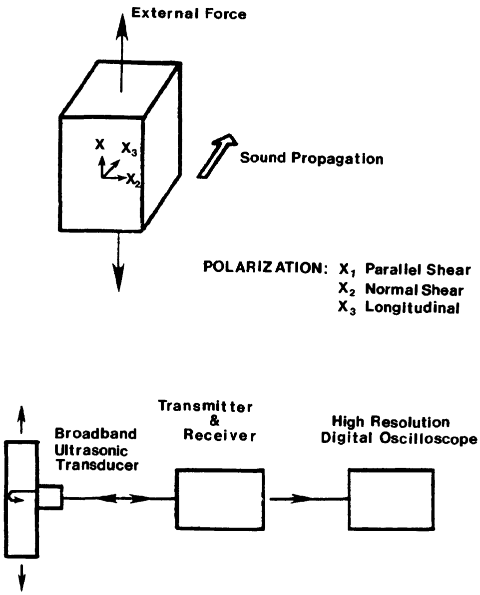

Fig. 2. Schematic of an acouto-elastic measurement system under uni-directional compression and tension.

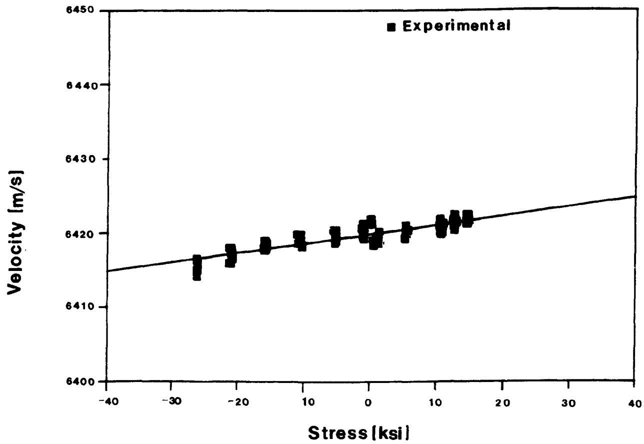

Fig. 3. Longitudinal wave velocity as a function of applied stress in 6061-T6 aluminum.

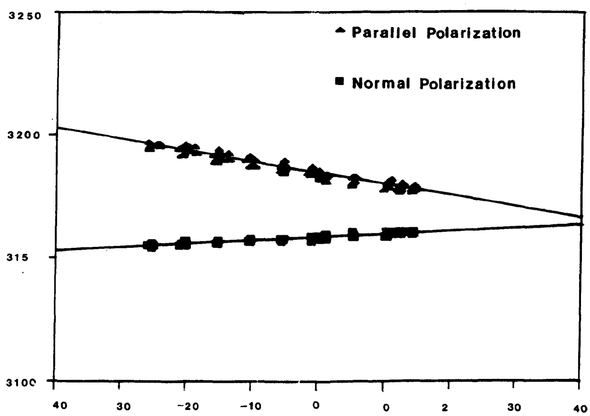

Fig. 4. Shear wave velocities as a function of applied stress in 6061-T6 aluminum.

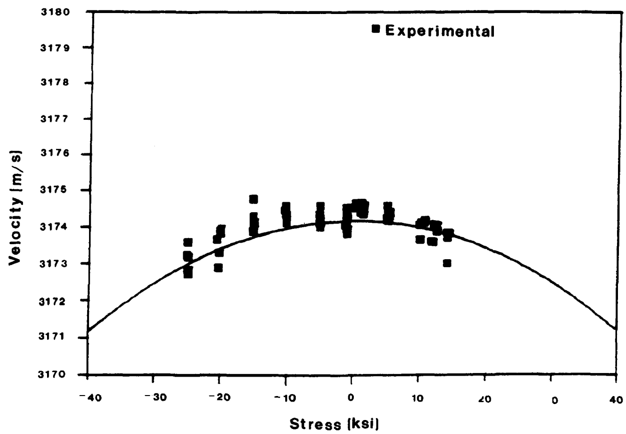

Fig. 5. A 28° polarized shear wave velocity as a function of applied stress in 6061-T6 aluminum.

"PURE" SHEAR STRESS TECHNIQUE

It has been pointed out already that the stress dependent velocities under pure shear stress have only stress square dependent terms because of symmetry [5]. This can be demonstrated by Fig. 6 where the sample is shown at zero, some positive and some negative shear stress, τ . Since

$$C(\tau) = C(-\tau),$$

using Eq. 1 one obtains

$$C(\tau) = C_0 + C_1\tau + C_2\tau^2 \quad \text{(for positive stress)} \tag{15}$$

$$C(-\tau) = C_0 - C_1\tau + C_2\tau^2 \quad \text{(for negative stress)} \tag{16}$$

which implies that the C_1 term has to be equal to zero because of Eq. 14.

To produce a pure shear stress for acousto-elastic measurements is not an easy task. We have used a simple configuration shown in Fig. 7a as a possible solution. A sample is held fixed at one end and a longitudinal transducer is mounted to the sample for velocity measurements. The sample is bent by applying loads in either upward or downward directions. Velocity is measured as a function of the applied load. It is realized that in addition to the shear stress there are also normal stresses present. But while there may be tension at a lower layer in the sample (when the material is pushed upward), the upper layer is compressed at the same time. Hence the integrated value of compression and tension will produce zero normal stress leaving only a "pure" shear stress on the sample. The received echo is shown schematically in Fig. 7b indicating only a second order effect. Whether the bending is upward or downward, the direction of the velocity shift is the same (decrease from zero stress value).

DETECTION OF WEAK BOUNDARY LAYERS IN ADHESIVES

One adhesive failure model assumes that the failure in an adhesive joint takes place in a very thin layer near the adhesive-adherend interface. This layer is known as Weak Boundary Layer. We would like to demonstrate that second order nonlinearity measurements are more sensitive to detect this W.B.L. than linear ones using the above described "pure" shear stress technique.

We modelled the adhesive layer by a plastic plate of a few millimeters thickness and the W.B.L. by an epoxy layer of 7% in the plastic plate of the same thickness. (In reality the W.B.L. may be less than 1% of the total adhesive thickness). The geometry of the two samples is shown in Fig. 8. We have carried out linear measurements by measuring the shear velocity of the two samples and found a 2.6% reduction of the velocity in the sample with the Weak Boundary Layer. The second order nonlinearity parameter showed a 600% variation between the two samples. Destructive shear stress showed that the shear strength is reduced by 300% when the

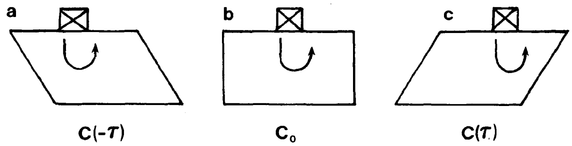

Fig. 6. Schematic diagram of pulse-echo velocity measurement in an undeformed solid (b) and in a solid with "pure" shear deformation (a) and (c).

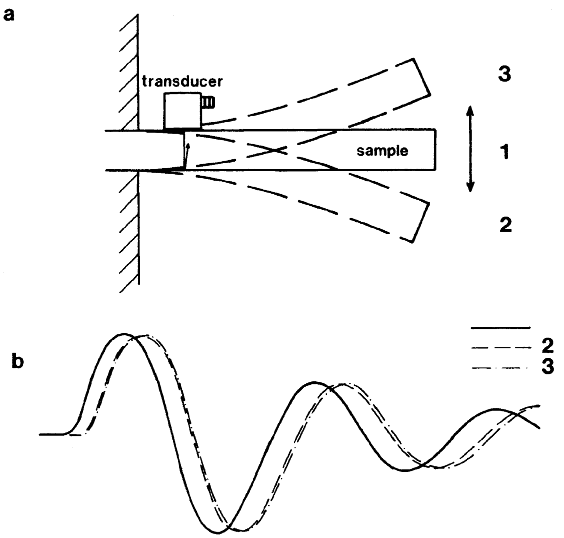

Fig. 7. (a) Geometrical configuration to produce "Pure" shear stresses, (b) Received echo.

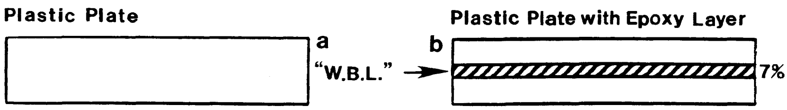

Fig. 8. Geometry of an (a) adhesive layer and (b) adhesive layer with W.B.L.

W.B.L. is present. One could easily measure second order nonlinearity variation between the two samples with this technique at a much lower load, perhaps at 10-15% range and detect a real weak boundary layer in the adhesive.

CONCLUSIONS

In this paper it was pointed out that second order acoustic nonlinearities may provide additional information in materials characterization. Two acousto-elastic measurement techniques were suggested to measure second order nonlinearities from the stress dependent behavior of the sound velocity. The first technique is based on minimizing the effect of shear wave "birefringence" (used to measure first order nonlinearities) due to stress induced anisotropy at an optimized polarization angle. At this polarization angle second order nonlinearities were measured in aluminum. The second order acousto-elastic constant is about 1% of the first order one. The second technique to measure second order nonlinearity is based on acousto-elastic measurement under an applied "pure" shear stress. Because of symmetry the stress dependent velocity depends on the square of the applied stress with only second order coefficients. We have also demonstrated on a model experiment that second order nonlinearity measurements are much more sensitive to detect the so-called weak boundary layer in an adhesive joint than linear measurements.

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